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The Impact of Network Structure on the Perturbation Dynamics of a Multi-agent Economic Model

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Abstract. Complex adaptive systems (CAS) modeling has become a common tool to study the behavioral dynamics of agents in a broad range of disciplines from ecology to economics. Many modelers have studied structure's importance for a system in equilibrium, while others study the effects of perturbations on system dynamics. There is a notable absence of work on the effects of agent interaction pathways on perturbation dynamics. We present an agent-based CAS model of a competitive economic environment. We use this model to study the perturbation dynamics of simple structures by introducing a series of disruptive events and observing key system metrics. Then, we generate more complex networks by combining the simple component structures and analyze the resulting dynamics. We find the local network structure of a perturbed node to be a valuable indicator of the system response.

Keywords: Structure, Perturbation, Network, Complex Adaptive System

1 Introduction

The increase in computing power seen in the past ten years has made agent-based modeling a viable option for studying complex systems. Recent work has begun to show the importance of network structure in the operation of complex adaptive systems (CAS), including how a system can be influenced using driver nodes [Liu et al. 2011]. Other research has focused on how structure can affect a diffusion algorithm as it propagates through a system [Ghoshal et al. 2011]. Researchers have also explored the implications of community structures in a network [Karrer et al. 2008]. While there is abundant research on CAS with a variety of structures, there has not been a systematic study of whether basic structural features could account for qualitative behavioral properties in large networks. This is an important question because structure has the potential to influence the robustness of a network. For

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example, supply networks and power grids losses due to perturbations could potentially be decreased by simply altering node connections. The inherent complexity of these systems makes analytical development of a perturbation theory difficult.

In this study we begin to explore dynamics within network structures resulting from perturbations. We compartmentalize complex networks into simple component structures whose dynamics are simply defined. We combine these compartmentalized networks and analyze their responses to perturbations. This experimental approach to system response provides valuable generalizations about complex networks.

1.1 Model Formulation

We study network structure and system dynamics using a configuration of the Interacting Specialist Model developed at Sandia National Laboratories [Beyeler et al. 2011]. In the interest of space, the model formulation is omitted but is rigorously defined in Beyeler et al. 2011. This model represents complex adaptive systems using coupled nonlinear first-order differential equations to describe the behavior of autonomous agents. The agents (or entities) must store, consume, and produce resources to maintain viability and competitiveness in their environment. The agents maintain their stability through a series of discrete interactions with markets, which create exchange pathways between agents. These interactions are facilitated through a money resource.

The model consists of a set of entities arranged in a hierarchy. Entities can be grouped into sectors, each of which is a collection of agents that produce and consume the same resources. Markets mediate transactions between sectors. A collection of sectors and markets makes up a Nation State.

Entities interact by joining a market and bidding to buy or sell resources. Consumers and producers are matched via a double auction. Entities make decisions about market transactions based on the entity health, resource reserves, and money levels. Health is defined as a scalar function that follows an agent's consumption with respect to a nominal consumption rate. Health abstractly represents a measure of an entity's success in a dynamic and competitive marketplace.

To study the dynamics of the model, we introduce perturbations and observe the system response. We simulate disruptive events by removing a certain percentage of an entity's produced resource in random events that occur with a defined frequency and duration. The resource is removed from the entity's production tank, preventing it from being sold to accrue a profit. This method can be used to represent a range of perturbation types, from an event analogous to a pipe bursting to smaller but more frequent perturbations, such as a 1% loss every time step, which simulates a leak in a pipe. This gives us considerable control over the perturbations we introduce into the model.

2 Structural Dynamics of Simple Networks

We would like to understand the dynamics of a complex network, such as an economy. Unfortunately, the complex feedback patterns created by a network of business relationships make it difficult to resolve causation. Thus, to make sense of the dynamics, we start by characterizing basic structures. These structures are idealized endpoints along axes of topological features commonly used to describe networks, such as path length and degree of connection. We use six sectors in each structure, and one entity in each sector. A connection from one sector to another means that a resource produced in the first sector is consumed in the second. We then consider a structure that superimposes several of these component structures and observe the resulting dynamics.

2.1 Fully Connected Networks

A fully connected network is defined as a network in which every node is connected in two directions to every other node in the system. This network is symmetrical and robust. Any perturbation will quickly reach every node, but because of the high connectivity, the impact is shared among several nodes and the system can cope with larger shocks.

Figure 1 illustrates the fully connected network and the response when node F is perturbed by a standard amount, removing 100% of its produced resource stores. By analyzing the responses at each node, we can characterize the nodal interactions very well.

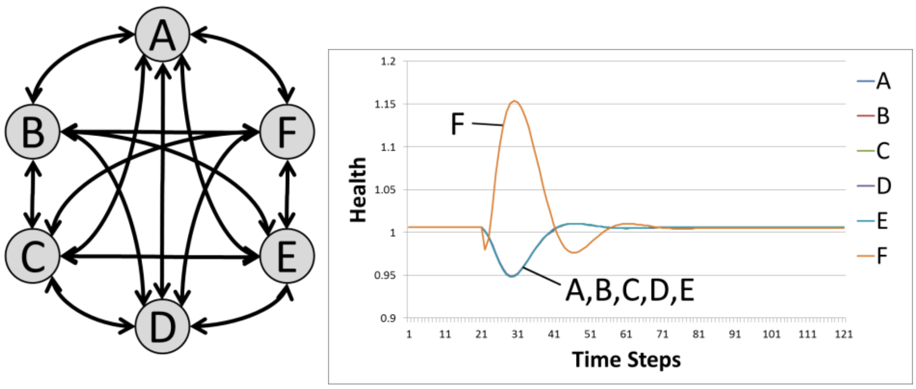


Fig 1. A fully connected network and the perturbation response when node F is shocked

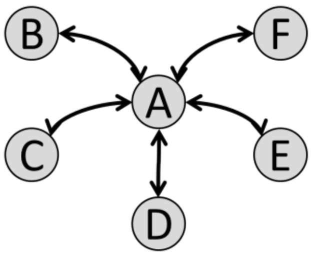
When node F is perturbed there is no product to sell and it immediately stops making money. As F lowers its consumption to preserve its money reserves, its health level is directly decreased. Because F's product is no longer available, all other sectors also begin to experience health deficits. Nodes A, B, C, D, and E see identical health trajectories because they are symmetric and identical.

Once F begins to produce again, it has a product that is very scarce with large demand. This causes the price to of F's product to spike, bringing in large profits, which spurs consumption leading to a large rise in health. Meanwhile, the other sectors are competing for a scarce resource at premium prices, causing them to consume less and continue to decrease in health. This behavior continues to a point at which F's product is overabundant, causing a trend reversal as the sectors readjust their consumption to changing prices. The health of the sectors then oscillates with a certain damping ratio until the system reaches a steady state again.

There are several notable features about this response. The oscillatory recovery response is a nontrivial trait of the fully connected network. Also, the perturbed node sees a maximum health deviation that is three times the maximum deviation in any other sector. Most importantly, the perturbed node sees a net health gain compared to the other sectors, which see a net health loss. This is similar to the competitive exclusion principle shown by Beyeler (2011). The perturbed sector is able to exploit the scarcity because our model has a fixed demand. In other systems, the market may have substitute goods and price would not spike.

2.2 Hub Networks

A hub network is asymmetrical, having a central node which consumes resources from every other node while also producing a resource that every other node requires. Although the vitality of this network depends upon the central node, the periphery nodes can also have a significant effect on the structure. Any perturbation quickly travels to the central node and then disperses through the rest of the network. It represents an extreme case of heterogeneity in connections.



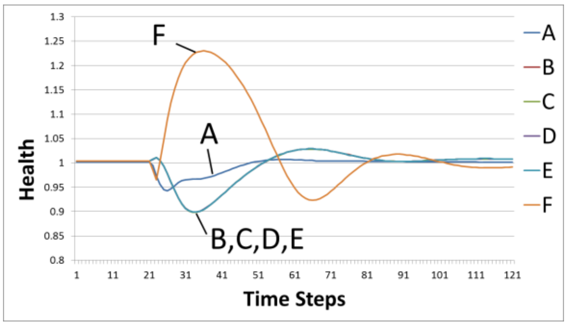


Fig 2. A hub network and a perturbation response for a shock on node F

The response to a standard, 100% removal of a produced resource from a peripheral node is remarkably similar to the response of a fully connected network. The hub network exhibits three responses corresponding to the perturbed node (F), the center node (A) and the periphery nodes (B to E). The maximum health deviation is larger than the response observed in the fully connected structure.

The behavior becomes more interesting when we perturb the center node in the network.

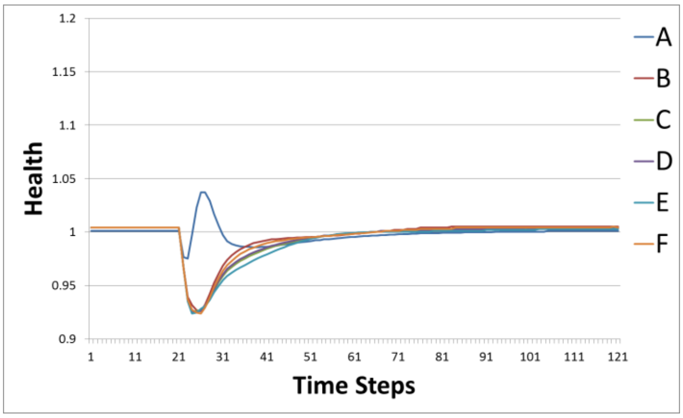


Fig 3. A perturbation response for a hub network with the center node shocked

When a standard perturbation removes 100% of the center node's produced resource, the magnitude at the perturbed node and the recovery time are significantly reduced compared to the same perturbation introduced into the periphery nodes. This counterintuitive result shows that the local structure of the perturbed node in the hub network controls price spikes through negative feedbacks. The perturbed central node begins to see a health rise due to the price spike for its produced resource. The periphery nodes see a health decline that mirrors the perturbed node's increase. However, the hub structure causes the center node to be critically dependent on the periphery nodes. As the peripheral nodes decrease in health, they decrease production and the perturbed central node's consumption demand is not met by the other sectors production supply, due to their low health. This limit on consumption effectively caps the perturbation response.

2.3 Circular Networks

A circular network is made up of several nodes connected linearly in a circular pattern. This network is symmetrical and offers significant buffers to perturbations. Shocks must travel linearly through every node, taking more time to reach every node in the system. This structure has the longest path length and the minimum connection degree of any symmetrical network. The drawback to these features is that the magnitude of each perturbation is passed through each node, which can push fragile nodes to their death.

A circular network has a very distinct perturbation response. Figure 4 shows the response from removing 100% of node F's produced resource. Unlike the response seen in other network structures, there is no significant health gain in the perturbed node following this disruption. Instead, the node that consumes the perturbed node's resource sees a significant health loss. The two closest upstream nodes generally experience a health gain, with a phase lagging behind the perturbed node, but node C, sees a health loss. Node C is directly opposite from the perturbed node on this circular network: positive and negative responses ripple towards it from opposing directions. The negative ripple dominates and the node sees a net health loss.

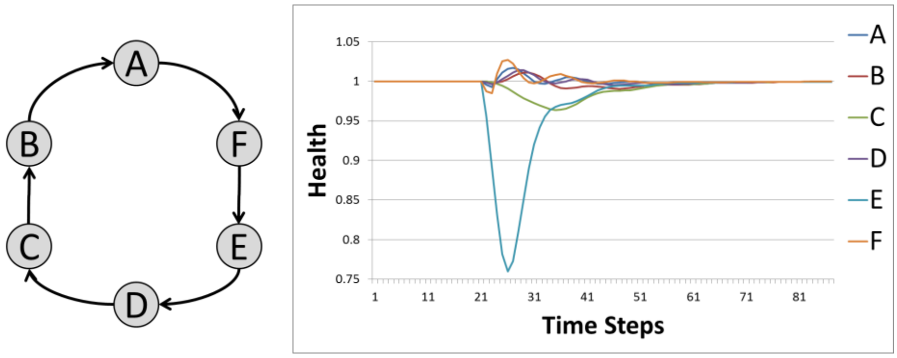


Fig 4. A circular network and a perturbation response for a shock on node F

3 Combined Networks

In order to understand the dynamics of complex networks, we combine several simple component structures in a new structure which is shown in Figure 5.

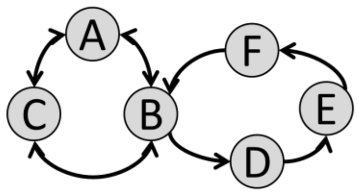


Fig 5. A fully connected network combined with a circular network

This structure consists of two distinct parts. Nodes A, B, and C make up a fully connected network. Nodes B, D, E and F make up a circular network. Node B is a critical node that connects these two structures.

Perturbing each node will produce a unique system perturbation response since the structure is asymmetric. We expect the nodal response for A and C to be similar to nodal responses for a fully connected structure, and the nodal responses for B, D, E, and F should display circular features. This is an approximation, since the combination of structural components adds new feedbacks that cause the system dynamics to change. Generally, for this combination, we find that the component circular structure retains many qualitative and quantitative features of its dynamics, while the fully connected network sees a few distinct changes. In this regard and for this combination, we can say that the behavior of the circular structure is more robust to structural combination.

We can be more precise by observing that the system dynamics change depending on what node is perturbed. When nodes far upstream are shocked (node D), the entire structure behaves similarly to a circular network. Conversely, when we perturb nodes close to supplying the transition node B, the entire system begins to behave with fully connected network dynamics.

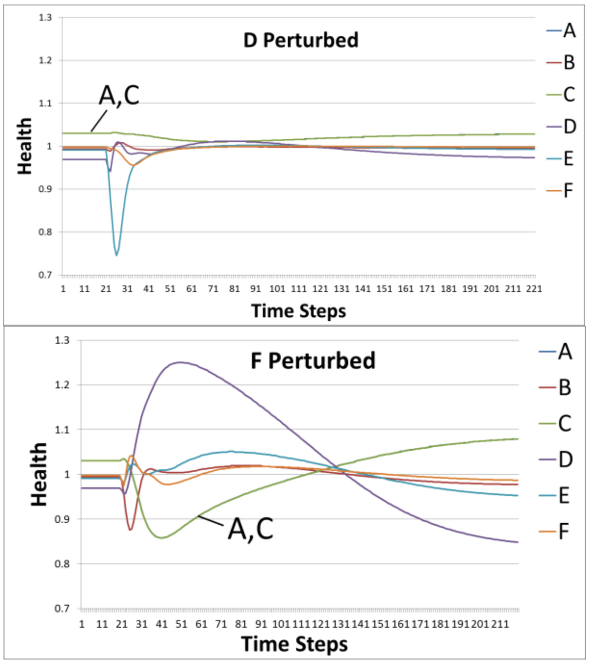


Fig 6. Note that perturbations to node D have a response similar to a characteristic circular structure, while perturbations introduced at node F display features of a fully connected network

Since the system perturbation response varies as the perturbed node changes, the structural components immediately downstream of a perturbation are most critical to the system's response. This makes sense, since health is tied to consumption. Perturbations disrupt downstream consumption rates, which controls the perturbation response of the system.

To explore the differences between the component structures more rigorously, we can analyze individual node responses. The most interesting node is the transition node (B). It is unclear how the component dynamics of the full and circular structures will interfere at this point. When we perturb the transition node, the response models the circular structure. The shock hits the downstream node (D) the most severely, and the perturbed node does not see drastic net gain in health observed in a fully connected system. The health gain that ripples through the circular structure with some phase lag is clearly seen, and the characteristic fully connected network response of a significant health gain is only seen by node D, although it is muted.

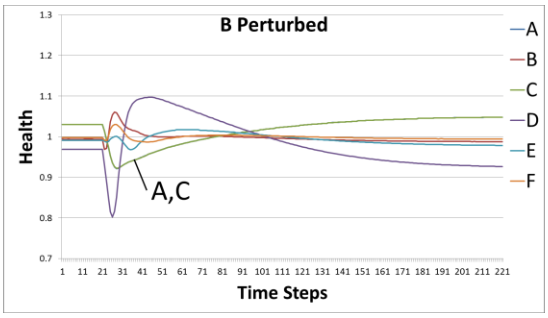


Fig 7. A combined fully connected network and circular network, where the transition node is perturbed

This combined network provides interesting insight into structure's affect on perturbation responses. We see that downstream structure is a primary factor in determining a response. Also, we see circular structures retain their dynamics more effectively than a fully connected structure. Most importantly, we see that structural features are not additive and complex network structure dynamics cannot easily be analytically predicted.

4 Conclusion

Studying structure's affect on system dynamics presents many new challenges. By characterizing simple component structures and observing competing dynamics in combined structures, we can gain insight into to how these networks interact. The results presented here suggest that characterization and combination of sub-structural features will be inadequate and yield misleading indicators of the system's response to perturbations. We expect that system dynamics will become more complex as network structure becomes more complex. However, some general rules may be forthcoming. For example, perturbations to peripheral nodes may produce much larger responses than perturbations to central nodes as we found in the simple hub structure. It is the search for these general rules towards which we must focus future work to understand the dynamics within complex networks made of economic agents.

References

1. Liu, Y.-Y., Slotine, J.-J., Barabasi, A.-L.: Controllability of complex networks. Nature 43, 123-248 (2011)
2. Ghoshal, G., Barabasi, A.-L.: Ranking stability and super-stable nodes in complex networks. Nature Communications 2, 1-7 (2011)
3. Karrer, B., Levina, E., Newman, M.E.J.: Robustness of community structure in networks. Phsycial Review E 77, 046119 (2008)
4. Beyeler, W.E., Glass, R.J., Finley, P.D., Brown, T.J., Norton, M.D., Bauer, M., Mitchell, M., Hobbs, J.A.: Modeling systems of interacting specialists. 8TH International Conference on Complex Systems. (2011)